MATHEMATICAL MODEL OF TIP-HYPHA ANASTOMOSIS AND DICHOTOMOUS BRANCHING WITH TIP DEATH DUE TO OVERCROWDING

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ABSTRACT. In this paper, we investigated the case of fungi growth when more types of dichotomous branching are combined. These consume all energy so Lateral branching, Tip-Hypa Anastomosis, Tip-anastomosis, and Tip death due to overcrowding species. We use mathematical models such as partial differential equations (PDEs) to illustrate biological phenomena of various types. We need some time for fungi to grow; that is true. We will use the numerical solution to obtain an approximate solution for this system. This solution's steps are stationary, phase, and traveling states. Solution and determination of the initial condition We will use the code to recognize the behavior of the types.

JEL Classification: example
D02, O17, P31

Keywords: Lateral branching, Tip-Hypa Anastomosis, Tip-tip anastomosis, Tip death due to overcrowding

Introduction

During the growth stages of a specific type of fungi, a group of any type can be expressed. To facilitate discussion of these types, abbreviated symbols for each type are used, as shown in table (1), which describes some biological types, each of which is mathematically analyzed and given an explanation and a description of the parameters. In this paper, we will combine various types of fungi [1, 2].

1. Literature review

Mathematical Model
We will study a new kind of the fungal branching with fungal death is, dichotomous branching with Tip-hypa anastomosis and Lateral branching, Tip death due to overcrowding (F+H+W+X). The table below shows these kinds:
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<table>
<thead>
<tr>
<th>Biological Kind</th>
<th>symbol</th>
<th>version</th>
<th>parameters</th>
<th>Parameters Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tip-hypha Anastomosis</td>
<td>H</td>
<td>$\sigma = -\beta 2np$</td>
<td>$\beta 2$</td>
<td>$\beta 2$ is the rate of Tip reconnections per unit length hypha per unit time</td>
</tr>
<tr>
<td>TiP death due to overcrowding</td>
<td>X</td>
<td>$\delta = -\beta 3p^2$</td>
<td>$\beta 3$</td>
<td>$\beta 3$ is the rate at which overcrowding density limitation</td>
</tr>
</tbody>
</table>

Table (1): Displaying the Biological kind, its symbol, versions, and descriptions for these parameters

1.1 FHXW Branching Type Without Hyphal Death

In this section, we will study a new type of fungal growth without hyphal death that mean

$\delta(p, n) = F + H + W + X$  \hspace{1cm} (1.1)

Where,

F: Lateral branching,
H: Tip-Hypha Anastomosis,
W: Tip-tip anastomosis,
X: TiP death due to overcrowding.

The model system for F HXW is

$$\frac{dp}{dt} = n *$$

$$\frac{dn}{dt} = \frac{\alpha 2v}{\gamma 2} p * - \left( \frac{\beta 2vm}{\gamma 1} \right) n * p * - \left( \frac{\beta 1n}{\gamma 1} \right) n * 2 - \left( \frac{\beta 3vp}{\gamma 1} \right)p^2$$  \hspace{1cm} (1.2)

After dropping stars, choosing $n = \frac{\alpha 2}{\beta 2}$ and $p = \frac{\alpha 2\beta 1\gamma 1}{\beta 2\beta 3v}$ the system (1) becomes

$$\frac{dp}{dt} = n$$

$$\frac{dn}{dt} = \frac{\alpha 2v}{\gamma 1} + \alpha p (1 - n) - \beta (n^2 + p^2)$$  \hspace{1cm} (1.3)

Where $\alpha = \frac{\alpha 2v}{\gamma 1}$ and $\beta = \frac{\alpha 2\beta 1}{\beta 2\gamma 1}$

1.2 The Stability of Solution

In this section, we will illustrate the stability of system (1) as follows:
n=0
αp(1-n)-β(n^2 + p^2)

the solution of these equations, we will find values of (p, n), the steady state are: (0, 0) saddle point, and (α, 0): unstable spiral, see Fig (1)

Fig (1) The (p,n)-plane: note that a trajectory connects the saddle point (0, 0) to the stable spiral (α, 0) where α = β = 0.33. Solutions are produced using MATLAB pplane7

1.3 Traveling Wave Solution

Hence we seek traveling wave solutions to (1.2). A mathematical way of saying this that we seek solutions of the form

P(x, t) = P(z)
n(x, t) = N(z) \quad (1.5)

Where z = x−ct. P(z) and N(z) represent density profiles, and c can be interpreted as the colony edge's propagation rate. For these to be biologically meaningful, we require P and N to be bounded, non-negative functions of z. Then P(x, t) and n(x, t) is a traveling wave that moves at a constant speed c in the positive x-direction if c is positive. Clearly if (x − ct) is constant, so are P(x, t) and n(x, t). It also means the coordinate system moves with speed c. The wave speed c
generally has to be determined. The dependent variable $z$ is sometimes called the wave variable. When we look for traveling wave solutions of an equation or system of equations in $x$ and $t$ in the form (1.5), we have 

\[ \frac{dP}{dz} = \frac{1}{c}[N] \]

\[ \frac{dN}{dz} = \frac{1}{c-1}[\alpha p(1-N) - \beta(N^2 + P^2)], \quad c \neq 1, \quad -\infty < z < \infty \]  

(1.6)

1.4 Stability of Traveling Wave Solution

The steady state of system (1) are $(p, n) = (0, 0)$, and $(\alpha, 0)$ here $(0, 0)$ is stable node and $(\alpha, 0)$ is saddle point for all $c$ positive. See figure (1.2). This information will help us to determine the initial condition of MATLAB pplane7 code,

Fig(2) The $(P, N)$-plane: note that a trajectory connects the unstable spiral $(\alpha, 0)$ to the saddle point $(0, 0)$ for all $c$ positive and $\alpha = 1$. Solutions are produced using MATLAB pplane7.

1.5 Numerical Solution

To solve this system (3), we will use pdepe code in MATLAB. To show the behavior branch and tips, see Fig (3), which represents the initial
condition branch (p) and tips (n),

Fig(3) the initial condition of (3), the solution to the system (3) with the parameters α = 1 and β = 1. Solutions are produced using pdepe in MATLAB.

Fig(4) illustrates the traveling waves at the suitable time (t): Solution to the system (1.3) with the parameters α = 1 and β = 1, for time t = 20, ..., 200, where blue line represented tips (n). Solutions are produced using pdepe in MATLAB.
Fig(5): illustrates the traveling waves at the suitable time (t): Solution to the system (1.5) with the parameters $\alpha = 1$ and $\beta = 1$, for time $t = 20, ..., 200$, where red line represented branches (p). Solutions are produced using pdepe code in MATLAB. From this operations, we get the relationship between traveling wave solution (c) and parameters $\alpha, \beta$ where $\alpha$ increasing the traveling wave solution (c) is increasing, and $\beta$ increasing the traveling wave solution (c) is decreasing, see Fig(5,4)

Fig(6) Illustrates the relation between the value $\alpha$, and waves speed $c$.

Conclusion

From the above results, Fig (6), We conclude that the traveling wave $c$ increases whenever the values of $\alpha$ increase for time $t$. Also, we know the value of $\alpha = (\alpha 1)/\gamma$, and We observe that it is directly proportional to $\alpha 1$ (the number of tips produced per unit time) and inversely proportional to $\gamma$. (The rate constant for the hyphal autolysis). That is, from a biological standpoint, growth increases whenever it increases.
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