EFFECT HYPHAL DEATH ON (F-X-W-D) BRANCHING TYPE WITH ENERGY

ABSTRACT: In this paper we studied the case of growth of types of fungi when mix three types Lateral branching, Tip death due to overcrowding, Tip-tip anastomosis. These types are consumed all the energy, this biological phenomenon represented as mathematical model as (PDEs). We need some time which is fact for growth of fungi. Solution of system depended on numerical solution and this solution gives approximation solution. Some steps on this solution as steady states, phase plane, travelling wave solution and using code (pplane7, pdepe) to solve it when determent the initial condition after that we shows the behavior of growth of fungi.

JEL Classification: D02, O17, P31

Keywords: Lateral branching, Tip death due to overcrowding, Tip-tip anastomosis, Hyphal death.

Introduction

We developed new models for the growth of fungal mycelia. Partial differential equations reflect the interaction of biomass with the underlying substrate are the best choice at this size. These models have a complicated mathematical structure including parabolic and hyperbolic elements. As a result, their analytic and numerical features are complex, and a group of any number of types can be expressed during the growth stages of a specific fungus. To make it easier to explain these types, abbreviated symbols for each type are used, like in table (1), which shows several biological types that have been mathematically examined and have been given an explanation and a description of the parameters. We shall blend various species of fungi in this paper.
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<table>
<thead>
<tr>
<th>Biological type</th>
<th>Symbol</th>
<th>Version</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral branching</td>
<td>F</td>
<td>( \delta = \alpha_2 \rho )</td>
</tr>
<tr>
<td>Tip death due to overcrowding</td>
<td>X</td>
<td>( \delta = -\beta_3 \rho^2 )</td>
</tr>
<tr>
<td>Tip-tip anastomosis</td>
<td>W</td>
<td>( \delta = -\beta_1 n^2 )</td>
</tr>
<tr>
<td>Hyphal death</td>
<td>D</td>
<td>( d = \gamma_1 \rho )</td>
</tr>
</tbody>
</table>

Table 1: Illustrate branching, Biological type, Symbol of this type and version.

1. **Mathematical Model**

We will study a new type of branching of fungal growth with hyphal death and Consumption of whole vegetarian food. We can call it energy \( \Psi(x) \), this energy function lies between one and zero as \( 0 \leq \Psi(x) \leq 1 \). Here if \( \Psi(x) = 1 \) that mean is the growth is very good if the fungi consume all the energy if \( \Psi(x) = 0 \) means if the grow die if it is not consume energy.

We can describe hyphal growth by the system below[2]:

\[
\begin{align*}
\frac{\partial \rho}{\partial t} &= n v - \rho \\
\frac{\partial n}{\partial t} &= -\frac{\partial (n v)}{\partial x} + e^{[\sigma(\rho, n)]} - \Psi
\end{align*}
\]

(1)

Where: \( \sigma(\rho, n) = \alpha_2 \rho - \beta_3 \rho^2 - \beta_1 n^2 \) and \( \Psi = 1 \). Then the system (1) becomes:

\[
\begin{align*}
\frac{\partial \rho}{\partial t} &= n v - \rho \\
\frac{\partial n}{\partial t} &= -\frac{\partial (n v)}{\partial x} + e^{[\alpha_2 \rho - \beta_3 \rho^2 - \beta_1 n^2]} - 1
\end{align*}
\]

(2)

2. **Non-dimensionlision and Stability**

Leah-Keshet (1982) and Ali H. Shuaa Al-Taie (2011) clear up how can put these parameters as dimensionlision less[6].

\[
\begin{align*}
\frac{\partial \rho}{\partial t} &= n - \rho \\
\frac{\partial n}{\partial t} &= -\frac{\partial n}{\partial x} + e^{[\beta n - \alpha (\rho^2 + n^2)]} - 1
\end{align*}
\]

(3)

Where: \( \alpha = \frac{\beta_1 \bar{n}}{\gamma} \) and \( \beta = \frac{\beta_3 \bar{n}}{\beta_3 \bar{v}} \)

Now, to find steady stste when take from system (3):

\[
e^{[\beta n - \alpha (\rho^2 + n^2)]} - 1 = 0
\]

\[
e^{[\beta n - \alpha (\rho^2 + n^2)]} = 1
\]

\[
ln e^{[\beta n - \alpha (\rho^2 + n^2)]} = ln 1
\]

\[
\beta n - \alpha (\rho^2 + n^2) = 0
\]

Then \( (\rho, n) = (0, 0) \), and \( (\rho, n) = \left(-\frac{\beta - \alpha}{\alpha}, -\frac{\beta - \alpha}{\alpha}\right) \)

Therefor we take the Jacobin of these equations

\[
J_{(\rho, n)} = \begin{bmatrix} -1 & 1 \\ \beta - 2\alpha \rho & -2\alpha n \end{bmatrix}
\]

Now, determent the eigenvalues as \( \lambda_i \); i=1,2
In this case we will take this point \((0,0)\) as saddle point and the point \(\left(\frac{\beta-\alpha}{\alpha}, \frac{\beta-\alpha}{\alpha}\right)\) stable spiral, for all \(\alpha, \beta > 0\) and \(\beta > \alpha\), see Fig (1) Using MATLAB pplane7[4,5].

![Figure 1: The \((n,p)\)-plane note that a trajectory connects the saddle point at \((0,0)\) and stable spiral at \(\left(\frac{\beta-\alpha}{\alpha}, \frac{\beta-\alpha}{\alpha}\right)\) for all \(\alpha=1, \beta=2\).](image)

3. Traveling Wave Solution

Now, we will discuss the traveling wave solution. Here we assume that: \(\rho(x,t)=P(z)\), and \(n(x,t)=N(z)\) where \(z=x-ct\), \(P(z)\) and \(N(z)\) are density profile and \(c\) rate of propagation of colony edge. \(P(z)\) and \(N(z)\) non-negative function of \(z\), the function \(\rho(x,t), n(x,t)\) are traveling waves and are moves at constant speed \(c\) in positive \(x\)-direction. Where \(c < 1\), \(\Psi(x)=1\), and \(\alpha=1\), and \(\beta=2\). To look for traveling wave solution of equations in \(x\) and \(t\) in the form (3)

\[
\frac{d\rho}{dt} = -c \frac{dP}{dz}, \quad \frac{dn}{dt} = -c \frac{dN}{dz}, \quad \frac{dn}{dx} = \frac{dy}{dz}.
\]

See [1], therefore the above equation becomes:

\[
\frac{dP}{dz} = \frac{-1}{c} [N - P],
\]

\[
\frac{dN}{dz} = e^{\frac{1}{1-c} [\beta N - \alpha (P^2 + N^2)]}, \quad c \neq 1, -\infty < z < \infty
\]

Then the steady state of equation (4) are:

\[
\frac{-1}{c} [N - P] = 0
\]

\[
\frac{1}{1-c} \beta N - \alpha (P^2 + N^2) = 0, \quad c \neq 1, -\infty < z < \infty
\]

To determine steady state of above system we get \((N,P)=(0,0)\), saddle point and \((N,P) = \left(\frac{\beta-\alpha}{\alpha}, \frac{\beta-\alpha}{\alpha}\right)\), unstable spiral, for all \(\beta > \alpha\), and \(c > 1\). See Fig(2) Using MATLAB pplane7[8].
Figure 2: The (N,P) plane note that a trajectory connects the from unstable spiral at \((\frac{\beta-\alpha}{\alpha}, \frac{\beta-\alpha}{\alpha})\) to saddle point at (0,0).

4. Numerica Solution

Now, we will solve the above system (3) using pdepe code in MATLAB, that is clear the initial condition start from 1 to zero for \(\rho\) and \(n\), figures below explains this. Illustrate figure (3) the initial condition and in figure (4) where the blue line represented tips (n), and in figure (5) where the red line represented branches (p), in figure (6) where the blue line represented tips (n) with the red line represented branches (p), for all \(\beta=0.5, \alpha=0.4\) and \(c=4.8481\) for time \(t=1,10,20,…,200\).
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Figure 4: The blue line represented tips (n).

Figure 5: The red line represented branches (p).
In this paper it was concluded the relationship between traveling wave solution $c$ and parameter $\alpha$ where traveling wave decreasing whenever the values of $\alpha$ increase [9, 10]. See Fig (7).

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>3.06</td>
<td>1.51</td>
<td>0.93</td>
<td>0.35</td>
<td>0.08</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 2: The relation between traveling wave $c$ and parameter $\alpha$ with taking $\beta=v=d=1$.

Now, we take relation between traveling wave speed $c$ and parameter $\beta$, where the traveling wave increasing whenever the values of $\beta$ increase. See Fig (8).
Figure 6: The relation between traveling wave c and parameter $\beta$.

Now, take relation between the traveling wave speed c and value v, such that the wave speed increasing when the values of v increasing. See Fig (9).

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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</thead>
<tbody>
<tr>
<td>c</td>
<td>0.44</td>
<td>1.51</td>
<td>4.84</td>
<td>6.25</td>
<td>8.91</td>
<td>10.53</td>
</tr>
</tbody>
</table>

Table 3: The relation between traveling wave c and parameter $\beta$ with taking $\alpha=v=d=1$.

Figure 6: The relation between traveling wave c and parameter $\beta$.

Table 4: The relation between traveling wave c and value v with taking $\beta=\alpha=d=1$.

<table>
<thead>
<tr>
<th>v</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>1.22</td>
<td>1.51</td>
<td>1.84</td>
<td>2.15</td>
<td>2.47</td>
<td>2.71</td>
</tr>
</tbody>
</table>

Figure 6: The relation between traveling wave c and value v.
Finally, we take relation between traveling wave $c$ and value $d$, when the wave speed decreasing when the values of $d$ increasin. See Fig (10).

<table>
<thead>
<tr>
<th>$d$</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>1.75</td>
<td>1.51</td>
<td>1.22</td>
<td>0.91</td>
<td>0.72</td>
<td>0.58</td>
</tr>
</tbody>
</table>

Table 5: The relation between traveling wave $c$ and value $d$ with taking $\beta=\alpha=\nu=1$.

Figure 6: The relation between traveling wave $c$ and value $d$.

Conclusion

We concluded frpm above results that the traveling wave $c$ decreasing whenever the values of $\alpha$ increase, see Fig (3), and we note the traveling wave increasing whenever the values of $\beta$ increase, see Fig (4), and we note the traveling wave increasing when the values of $\nu$ increase see Fig (5), finally the traveling wave decreasing whenever the values $d$ increase see Fig (6).

Sence ($\alpha = \frac{\beta_1 \bar{n}}{\gamma}$, and $\beta = \frac{\beta_3 \bar{n} \gamma}{\beta_3 \nu}$) therefore the growth rate is increasing with $\beta_1 \bar{n}$ while keeping $\gamma$ are fixed, while the growth is decreasing with $\gamma$ increases while keeping $\beta_1 \bar{n}$, and the growth rate is increasing with $\beta_3 \bar{n} \gamma$ while keeping $\beta_3 \nu$ are fixed, while the growth is decreasing with $\beta_3 \nu$ increases while keeping $\beta_3 \bar{n} \gamma$ are fixed.

References


